Question Bank for PG Course

অঙ্ক (Mathematics)

দশম (ক ১) পত্র (Paper - XA(i))

Advanced Differential Geometry : PGMT-XA(i)

1. Is the function $f: R \to R$ defined by $f(x) = x^3,\ \forall x \in R$ a homeomorphism from R to R?
2. What is the tangent vector to the curve $\gamma(t) = (2 + t, t, t^2)$ in R^3at the point $(1,1,1)$?
3. If $(x_1, \dots, x_n)$ is a local coordinate system in a neighbourhood U of p in the manifold M, then find the standard basis of the tangent space T_pM.
4. Find the integral curve $\gamma(t)$ of the vector field $\frac{\partial}{\partial x}$ in R^2.
5. For two smooth functions $f: M \to N \ and \ g: N \to K$,

   which of the followings is true?

   1. $(g \circ f)^* = f^* \circ g^*$
   2. $(g \circ f)^* = g^* \circ f^*$
   3. $(g \circ f)^* = f^* \circ g^* \circ f^*$
6. If $(x_1, \dots, x_n)$ is a local coordinate system in a neighbourhood U of p in the manifold M, then find the standard basis of the dual tangent space T_p^*M.
7. Compute the following

   $(2dx + dy) \wedge (dx - dy)$
8. What is the dimension of the following manifold?

   $\{(x, y, z) \in R^3 : x^2 + y^2 + z^2 = 1\}$
9. Which of the following condition is satisfied by linear connection ∇ on M?

   1. $\nabla_{fX} Y = f \nabla_X Y$
   2. $\nabla_{fX} Y = \nabla_X Y$
   3. $\nabla_{fX} Y = f \nabla_Y X$
10. "*Every Riemannain metric admits a unique Riemannian connection*"- is this statement true or false?
11. Which of the following expression is/ are true for Riemannian curvature tensor R

    1. $R(X,Y)Z + R(Y,Z)X + R(Z,X)Y = 0$
    2. $R(X,Y)Z + R(Z,X)Y = 0$
    3. $R(X,Y)Z + R(Y,Z)X + R(Z,X)Y = 1$
12. What is the scalar curvature of the Euclidean Space?

13. Which of the following expression is/ are true for Riemannian curvature tensor field R

1. $R(X,Y,Z,W) = -R(Y,X,Z,W)$

2. $R(X,Y,Z,W) = R(Y,X,Z,W)$

3. $R(X,Y,Z,W) = R(Y,Z,Z,W)$

4. $R(X,Y,Z,W) = -R(X,X,Z,W)$

14. If ω is a 1-form, then which of the followings is true?

$$1.\ \omega \wedge \omega = 1$$

$$2.\ \omega \wedge \omega = 0$$

$$3.\ \omega \wedge \omega = -1$$

15. What is the dimension of the tangent space of the manifold $= \{(x,y) \in R^2 : y > 0\}$?